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ORIGINAL

Optimizing Sequential Decisions: Enhancements to the Brickman Principle with Cumulative Punishment and Probability Adjustments

Optimización de decisiones secuenciales: mejoras al principio de Brickman con ajustes de probabilidad y castigo acumulativo

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ABSTRACT

Introduction: determining the optimal stopping point in sequential decision-making scenarios is crucial for maximizing rewards and minimizing costs. Traditional models like the original Brickman Principle often simplify this process by assuming fixed critical values and equal probabilities at each decision stage. These assumptions may not accurately reflect real-world complexities, where costs can be cumulative and probabilities variable.
Objective: this work seeks to enhance the Brickman Principle by including cumulative punishment elements and non-uniform probability distributions, therefore improving its capacity to accurately represent the intricacies of real-world decision-making.
Method: through a rigorous experimental study, we evaluate the impact of these modifications on optimal stopping rules and expected profits.
Results: in line with Prospect Theory's emphasis on loss aversion, the results reveal a distinct pattern of risk-averse behavior, with most participants choosing to stop sooner in the sequence to avoid growing fines. Furthermore, we saw substantial variation in both the termination points and anticipated earnings across participants, suggesting that individual disparities in risk tolerance and decision-making approaches are crucial in influencing results.
Discussion: this research contributes to the understanding of human decision-making processes and offers a robust framework for various applications, including financial investments, job offers, and purchasing decisions.

Keywords: Optimal Stopping; Sequential Decision-Making; Brickman Principle; Cumulative Punishment; Risk-Averse Behavior; Financial; Economy.

RESUMEN

Introducción: determinar el punto de parada óptimo en escenarios de toma de decisiones secuenciales es

crucial para maximizar las recompensas y minimizar los costos. Los modelos tradicionales como el Principio de Brickman original a menudo simplifican este proceso al suponer valores críticos fijos y probabilidades iguales en cada etapa de decisión. Estas suposiciones pueden no reflejar con precisión las complejidades del mundo real, donde los costos pueden ser acumulativos y las probabilidades variables. Este artículo propone una versión modificada del Principio de Brickman, que integra factores de castigo acumulativos y distribuciones de probabilidad no uniformes para capturar mejor los matices de la toma de decisiones práctica.
Método: a través de un estudio experimental riguroso, evaluamos el impacto de estas modificaciones en las reglas de parada óptimas y las ganancias esperadas.
Resultados: nuestros hallazgos revelan que el principio modificado proporciona un modelo más preciso y realista, que destaca los patrones de comportamiento de aversión al riesgo entre los individuos.
Discusión: esta investigación contribuye a la comprensión de los procesos de toma de decisiones humanas y ofrece un marco sólido para diversas aplicaciones, incluidas las inversiones financieras, las ofertas de trabajo y las decisiones de compra.

Palabras clave: Detención Óptima; Toma de Decisiones Secuencial; Principio de Brickman, Castigo Acumulativo; Conducta Adversa al Riesgo; Finanzas; Economía.

INTRODUCTION

Decision-making under uncertainty is a fundamental aspect of human behavior, particularly in financial contexts where individuals frequently choose between seeking better outcomes or securing the best available option.[1,2,3] The problem of when to stop in a sequence of offers to maximize expected value has been extensively studied through the lens of the optimal stopping theory. Formalized initially in the work of Chow, Robbins, and Siegmund,[1] the optimal stopping rule provides a framework for understanding the decision-making process in sequential choice problems.

Risk aversion, a key factor influencing decision-making, plays a significant role in optimal stopping. According to Prospect Theory,[2] individuals are typically more sensitive to potential losses than to equivalent gains, leading them to make more conservative choices to avoid loss. This behavior is often observed in financial decision-making, where risk-averse investors may prematurely sell winning investments to lock in gains or hold onto losing investments to avoid realizing a loss, as Shefrin and Statman[3] noted.

The present study extends the understanding of risk aversion in optimal stopping scenarios by incorporating a cumulative punishment factor. This modification reflects real-life financial decisions where prolonged exposure to risk incurs additional costs, such as transaction fees or opportunity costs. By examining how individuals adjust their stopping behavior in response to these cumulative costs, the study provides insights into the interplay between risk aversion and optimal stopping in sequential decision-making.

In this paper, we build upon the foundational principles of optimal stopping and Prospect Theory to analyze how cumulative punishment affects stopping behavior. The study utilizes a modified version of the Brickman Principle, integrating the impact of cumulative costs into the decision-making process. This approach provides a more realistic representation of financial decision-making, where costs increase with prolonged observation periods.

The findings of this study have practical implications for investors and financial advisors. Understanding how risk aversion and cumulative costs influence stopping behavior can help tailor investment strategies to align with individual risk preferences and optimize financial outcomes. By highlighting the role of cumulative punishment in decision-making, the study contributes to the broader literature on financial behavior and offers valuable insights for improving decision-making in uncertain environments.

In the following sections, we detail, the methodology used to derive the optimal stopping rules, present the results of our analysis, and discuss the implications of our findings in the context of existing literature on risk aversion and financial decision-making.

Review of Literature

Introduction to the Brickman Principle

The Brickman principle, initially formulated in the context of sequential decision-making, is a foundational concept in understanding optimal stopping rules. Traditionally employed in fields such as economics and operations research, this principle provides a framework for determining the point at which a decision-maker should stop and accept an offer rather than continue to seek potentially better options. The principle hinges on balancing the benefits of waiting for a better offer against the costs associated with continued waiting, including opportunity costs and the risk of receiving lower offers in the future.[4]

Historical Development and Applications

The Brickman principle's origins can be traced back to early work on optimal stopping theory and decision analysis. Early studies, such as those by Chow, Robbins, and Siegmund,[1] laid the groundwork by exploring the statistical properties of stopping rules in various contexts. These studies primarily focused on scenarios where the distribution of offers was known and decision-makers had perfect information.

Subsequent research expanded the application of the Brickman principle to more complex and realistic scenarios. For instance, dynamic programming approaches were developed to handle situations where decision-makers faced uncertainty and had to adapt their strategies based on observed outcomes.[5] These advancements made the Brickman principle more applicable to real-world decision-making processes, such as investment decisions, job search strategies, and consumer behavior.

The Brickman Principle in Risk and Uncertainty

One of the critical aspects of the Brickman principle is its ability to account for risk and uncertainty. Early applications often assumed a risk-neutral decision-maker, but later studies recognized the importance of incorporating risk preferences into the model. Research by Pratt[6] and Arrow[7] on risk aversion highlighted how individuals' varying tolerance for risk could significantly influence their stopping rules.

These insights led to the development of modified versions of the Brickman principle that explicitly incorporated risk aversion. These models adjusted the stopping rule to reflect the decision-maker's risk preferences, providing more accurate and personalized recommendations. This shift towards incorporating risk preferences marked a significant advancement in the applicability and relevance of the Brickman principle.

Limitations of Traditional Brickman Principle

While the Brickman principle has proven robust for optimal stopping analysis, it faces significant limitations in real-world scenarios. Two key limitations stand out:

Assumption of Known Distributions: Traditional models assume decision-makers know the probability distributions for potential offers. However, in practice, decision-makers often operate under uncertainty. They lack precise information about the offers' distribution, leading to suboptimal decisions if the model does not account for this uncertainty.[4]

Static Nature of Models: Traditional Brickman models do not adequately capture the dynamic aspects of decision-making. Real-world decisions involve cumulative factors, such as increasing costs or penalties for delayed choices. These dynamic elements were not reflected in the original formulations of the Brickman principle.[4] As a result, researchers have sought more flexible and dynamic models to address these limitations."

Advances in the Modified Brickman Principle

Recent advancements have addressed these limitations by introducing modifications to the Brickman principle. One significant modification is incorporating cumulative punishment factors, which reflect the increasing costs or penalties associated with delayed decisions. This modification allows the model to more accurately capture the dynamic nature of real-world decision-making processes.

Another important advancement is the integration of probability factors that account for the uncertainty in the distribution of offers. By considering the likelihood of various outcomes, these models provide a more realistic framework for decision-making under uncertainty. Research by Liu and Wei[8] demonstrated how incorporating these probability factors could enhance the accuracy and robustness of optimal stopping rules.

The present study builds on these advancements by integrating cumulative punishment and probability factors into the Brickman principle. This modified approach provides a more comprehensive and realistic analysis of decision-making behavior. The study offers valuable insights into risk-averse behavior and decision-making strategies by examining the optimal stopping rules and expected profits under this modified framework.

This study's findings align with the broader literature, highlighting the importance of considering immediate and long-term consequences in decision-making models. Integrating cumulative punishment and probability factors represents a significant step forward in the evolution of the Brickman principle, making it more applicable to diverse real-world scenarios. This research contributes to a deeper understanding of optimal stopping decisions and risk management by bridging the gap between theoretical models and practical applications.

The evolution of the Brickman principle from its traditional formulations to its modified versions reflects ongoing efforts to enhance the accuracy and relevance of optimal stopping models. Researchers have developed more robust and applicable frameworks by addressing the limitations of earlier models and incorporating dynamic and probabilistic elements. The present study continues this tradition by integrating cumulative punishment and probability factors, offering a more comprehensive analysis of decision-making behavior. As such, it contributes to the growing literature on optimal stopping theory and its practical applications in various fields.

Risk aversion is critical in decision-making processes, particularly in scenarios involving uncertain outcomes. This review explores the foundational theories and empirical studies on risk aversion, focusing on its implications for optimal stopping rules. The objective is to contextualize the current study within the broader literature on risk aversion and optimal stopping, highlighting key theoretical advancements and empirical findings.

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Theoretical Foundations of Risk Aversion

Risk aversion, a concept deeply rooted in economic theory, describes the preference for certainty over uncertainty. Pratt's[6] seminal work on risk aversion provided a formal definition and measurement, introducing the concept of the utility function to represent individual preferences. Arrow[7] further developed these ideas, emphasizing the role of risk aversion in economic behavior and decision-making under uncertainty.

Optimal Stopping Theory

The theory of optimal stopping deals with the problem of choosing a time to take a particular action based on sequentially observed random variables to maximize an expected payoff or minimize an expected cost. Chow, Robbins, and Siegmund[1] provided a comprehensive treatment of optimal stopping problems, laying the groundwork for subsequent studies. Their work highlighted the importance of understanding the underlying distribution of outcomes and the decision-maker's risk preferences.

Incorporating Risk Aversion into Optimal Stopping Rules

Incorporating risk aversion into optimal stopping rules requires a nuanced understanding of how individuals evaluate potential outcomes. Liu and Wei[8] explored this integration, demonstrating that risk-averse individuals tend to stop earlier than their risk-neutral counterparts. Their findings underscore the need to consider risk preferences when developing and applying optimal stopping rules.

Advances in Understanding Risk Preferences

Recent advancements in behavioral economics have provided deeper insights into risk preferences and their impact on decision-making. Kahneman and Tversky's[2] Prospect Theory revolutionized the understanding of risk by introducing the concept of loss aversion, where losses loom larger than gains. This theory has been instrumental in explaining why individuals often exhibit risk-averse behavior in uncertain situations.

Risk Aversion in Financial Decision-Making

Empirical studies have validated theoretical predictions about risk aversion and optimal stopping. Studies on financial decision-making have shown that investors' risk aversion significantly influences their portfolio choices and the timing of asset sales.[9,10] These findings align with the principles outlined in the theoretical literature, confirming that risk-averse individuals are more likely to adopt conservative strategies in uncertain environments. Experimental economics has provided robust evidence of risk aversion in various decision-making contexts. Holt and Laury[11] conducted experiments that quantified risk aversion levels among participants, showing substantial variation in risk preferences. These findings highlight the importance of accounting for individual differences in risk aversion when developing optimal stopping rules.

Real-world applications of risk aversion principles demonstrate their relevance across various domains. For example, in insurance markets, individuals' risk aversion drives the demand for insurance products to hedge against potential losses.[12] Similarly, in career decision-making, risk-averse individuals may prefer stable employment opportunities over entrepreneurial ventures, reflecting their preference for certainty.[13]

Behavioral biases, such as overconfidence and optimism, can interact with risk aversion, influencing decision-making processes. Odean[9] found that overconfident investors tend to underestimate risks and trade more frequently, often to their detriment. Understanding these biases is crucial for developing strategies that mitigate their impact on optimal stopping decisions.

Research has shown that gender differences can influence risk aversion, with women generally exhibiting higher levels of risk aversion than men.[14,15] These differences have implications for financial decision-making, career choices, and other areas where risk assessment is critical. Cultural factors also play a significant role in shaping risk preferences. Hsee and Weber[16] found that individuals from different cultural backgrounds exhibit varying levels of risk aversion influenced by societal norms and values. These cultural differences must be considered when applying optimal stopping rules in diverse populations.

The present study builds on this rich body of literature by examining the impact of risk aversion on optimal stopping decisions in a structured experimental setting. By incorporating a punishment factor for observing beyond a certain stage and adjusting the probability distribution of expected values, the study aims to provide a more nuanced understanding of how risk preferences influence stopping behavior. Using modified Brickman principles allows for a detailed analysis of the interplay between risk aversion, punishment, and optimal stopping decisions.

Risk aversion is critical in decision-making processes, particularly in scenarios involving uncertain outcomes. This review explores the foundational theories and empirical studies on risk aversion, focusing on its implications for optimal stopping rules. The objective is to contextualize the current study within the broader literature on risk aversion and optimal stopping, highlighting key theoretical advancements and empirical findings.

Experimental Evidence of Risk Aversion

Experimental economics has provided robust evidence of risk aversion in various decision-making contexts. Holt and Laury[11] conducted experiments that quantified risk aversion levels among participants, showing substantial variation in risk preferences. These findings highlight the importance of accounting for individual differences in risk aversion when developing optimal stopping rules.

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Cultural factors also play a significant role in shaping risk preferences. Hsee and Weber[16] found that individuals from different cultural backgrounds exhibit varying levels of risk aversion, influenced by societal norms and values. These cultural differences must be considered when applying optimal stopping rules in diverse populations. Understanding risk aversion is essential for policymakers and regulators, particularly in designing interventions to promote financial stability and consumer protection.

The present study by incorporating a punishment factor for observing beyond a certain stage and adjusting the probability distribution of expected values, the study aims to provide a more nuanced understanding of how risk preferences influence stopping behavior. The use of modified Brickman principles allows for a detailed analysis of the interplay between risk aversion, punishment, and optimal stopping decisions.

The literature on risk aversion and optimal stopping provides a robust theoretical and empirical foundation for understanding decision-making under uncertainty. The integration of risk preferences into optimal stopping rules is well-supported by both theoretical models and empirical evidence, highlighting the importance of considering individual differences in risk aversion. The present study contributes to this literature by offering a novel experimental approach to examining these concepts, providing insights that have practical implications for a wide range of decision-making scenarios.

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METHOD

This section outlines the methodology employed in our experimental study, including the design of the experiment, the process of data collection, and the analytical techniques used to interpret the results. The experiment simulated a sequential decision-making process where participants were presented with a series of offers, requiring them to decide at each stage whether to accept the current offer or wait for the next. The participants were 25 graduate students from Sri Ramakrishna College of Arts and Science (Autonomous), Coimbatore, India, recruited through voluntary participation. Each participant received a monetary incentive based on 10 % of the final value chosen, minus a cumulative penalty.

Key Elements of the Design

- *Offers*: A set of 20 predetermined offers was used, with the following values: [25, 94, 63, 37, 72, 18, 67, 42, 83, 55, 29, 78, 34, 91, 56, 12, 68, 45, 23, 87]. These values were generated using a random number generator with a uniform distribution between 1 and 100. The same sequence was presented to all subjects to maintain consistency across trials.
- *Cumulative Punishment*: A penalty for delaying decisions was applied after the 5th stage, increasing by a fixed amount (e.g., 5 units) with each subsequent stage. The exact penalty applied was calculated as:
  Cumulative Penalty= 5 × (Stage−5), forStage>5
  This reflects the increasing cost of delayed decisions.
- *Probability Distribution*: Offers were presented at each stage under the assumption of an even distribution, with a probability of 0,4 for receiving each offer. The same probability distribution was used throughout the experiment to simulate real-world randomness.

Procedure

1. Each participant was instructed to make decisions on whether to accept or reject offers at each stage, based on potential gains and penalties.
2. The stopping point for each subject—i.e., the stage at which they accepted an offer—was recorded.
3. The experiment was conducted in an isolated computer lab, with all instructions provided digitally. Participants were not allowed to consult others during the experiment.

Data Collection

Data were collected from a sample of 25 subjects who participated in the experiment. Each subject made sequential decisions based on the presented offers and the associated cumulative penalties. The stopping point for each subject was recorded, along with the profit or loss incurred.

- *Observed Stopping Points:* The stopping points recorded for the subjects were as follows: [1, 1, 0, 1, 19, 0, 1, 0, 10, 2, 1, 1, 8, 1, 1, 1, 1, 14, 1, 4, 8, 18, 1, 1, 1].
- *Profit Calculation*: The profit for each subject was calculated using the formula:

$$\textbf{Profit} = \textbf{Offer Value} \times \left(\frac{\textbf{Profit Percentage}}{\textbf{100}}\right) - \textbf{Cumulative Cost}$$

where the cumulative cost was applied only if the stopping point was beyond the 5th stage.

Analytical Techniques

Expected Profit Calculation

We calculated expected profits for each subject using a Monte Carlo simulation, leveraging the Metropolis-Hastings Markov Chain Monte Carlo (MCMC) method. We ran the MCMC simulations for 10,000 iterations per subject, which generated a distribution of possible stopping points and provided a robust estimate of the expected profit for each participant.

Optimal Stopping Rule

We derived the optimal stopping rule for each subject by averaging the stopping points obtained from the MCMC samples. This provided a benchmark for comparing actual stopping behavior with theoretically optimal decisions.

Risk Behavior Analysis

To assess risk behavior, we compared the observed stopping points with the MCMC-derived optimal stopping rules. We classified subjects as risk-averse if they stopped earlier than the model predicted, and as risk-tolerant if they delayed stopping despite potential penalties.

Interpretation of Results

The results of the analysis provided insights into the decision-making behavior of the subjects and the effectiveness of the modified Brickman Principle. Specifically, we examined the optimal stopping rules and expected profits to identify any patterns indicative of risk-averse behavior. The following sections discuss these results in detail, comparing the modified principle with the original and highlighting the implications for decision-making under uncertainty.

This study examined several key variables to assess their influence on decision-making in sequential scenarios. In this study, the offer values and cumulative punishment are the independent variables. The dependent variables include the stopping points, and the expected profits. We explored individual risk preferences as a moderating variable, which influenced the subjects' weighing of potential gains against the risk of cumulative penalties. We analyzed these variables to understand their interplay in shaping optimal stopping behavior and financial outcomes.

RESULTS

The optimal stopping points for each subject varied considerably, ranging from 5,29 to 12,98 stages. This variation suggests a variety of decision-making strategies influenced by the sequential offer values and the introduction of cumulative penalties. Subjects tended to stop relatively early in the sequence, with an average optimal stopping point of around 7. This early stopping behavior aligns with the tendency toward risk aversion, as discussed in Prospect Theory.[2] Risk-averse individuals are more likely to secure a smaller but certain gain, rather than continue waiting for higher offers, which carry the risk of incurring cumulative penalties.

The fixed sequence of 20 offers, ranging in values from 12 to 94, had a significant impact on the stopping points. The variability in offer values likely influenced the subjects, leading them to stop earlier when presented with a moderate or high offer, rather than waiting for a potentially better one. For example, participants who encountered early offers in the range of 83 or 87 were more likely to stop early, reflecting their preference for avoiding further risks.

Table 1. Summary of Optimal Stopping Rule for Each Subject

Subject	Optimal Stopping Rule
1	5,36181633
2	5,37379592
3	8,84197959
4	5,40420408
5	12,97683673
6	8,85502041
7	5,32987755
8	8,81816327
9	9,39320408
10	7,28563265
11	5,34787755
12	5,36863265
13	8,83161224
14	5,36055102
15	5,37234694
16	5,42495918
17	5,29010204
18	10,35893878
19	5,37271429
20	7,52418367
21	8,82814286
22	9,69573469
23	5,41453061
24	5,40638776
25	5,38191837

Source. Data collected by the researchers from an experimental study

Table 1 provides a summary of the optimal stopping rule for each subject in the study. The values indicate the specific point at which each subject is predicted to stop based on the collected data.

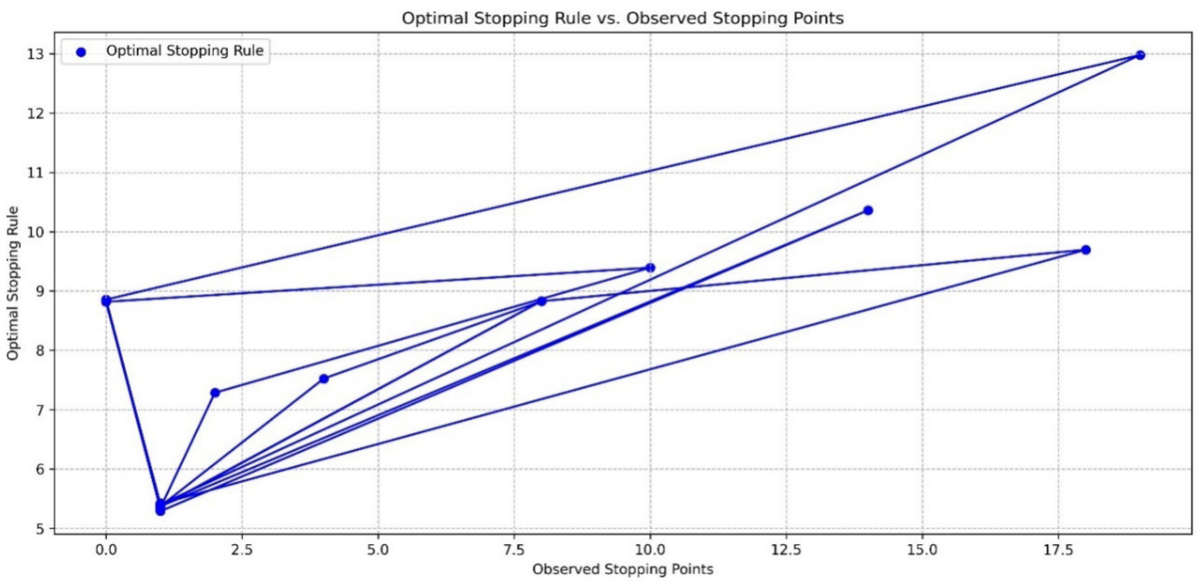

Figure 1. Comparison of Optimal Stopping Rule and Observed Stopping Points
Source. Data collected by the researchers from an experimental study

Figure 1 illustrates the relationship between observed stopping points and the optimal stopping rule. The graph shows multiple data series, each representing a different subject. As the observed stopping points increase, the optimal stopping rate generally trends upward, indicating a positive correlation between the two variables. This suggests that higher observed stopping points are associated with higher optimal stopping rates.

Table 2. Summary of Expected Profit for Each Subject

Subject	Expected Profit
1	76,67026327
2	76,66763673
3	50,57636735
4	76,41488571
5	67,09944082
6	50,68211224
7	76,62197755
8	50,5668898
9	50,77579796
10	60,13069796
11	76,62438776
12	76,41967959
13	65,22948571
14	76,55024082
15	76,55223469
16	76,31917959
17	76,92239184
18	55,59597347
19	76,5959
20	64,02272857
21	65,35130612
22	50,17539796
23	76,34286735
24	76,41769796
25	76,57323265

Table 2 presents the expected profit for each subject in the study. The values indicate the anticipated profit for each subject based on their respective data, with most subjects showing expected profits in the range of approximately 50 to 77 units.

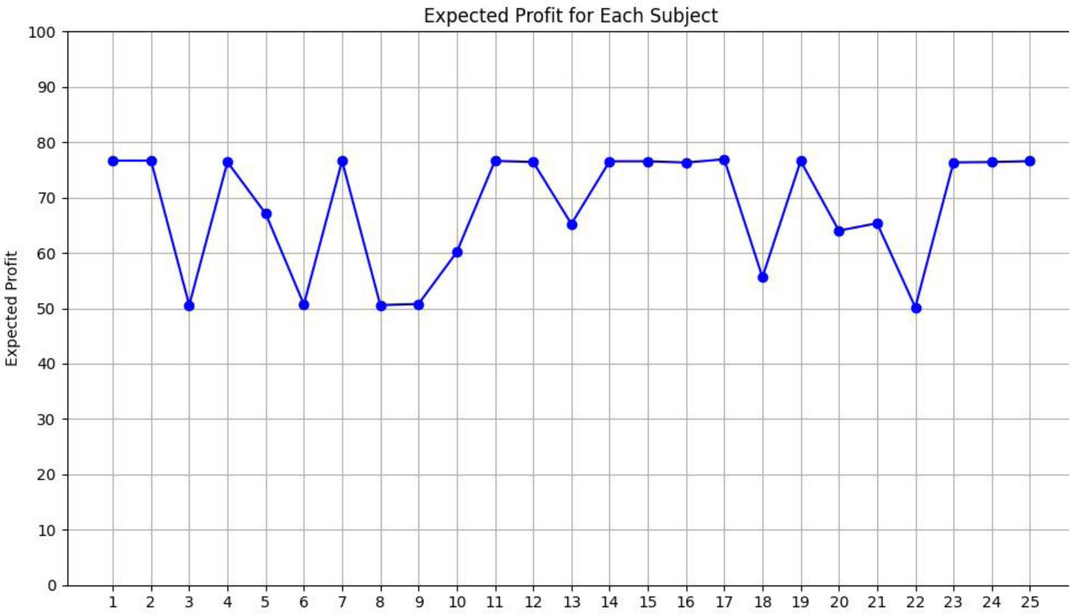

Figure 2. Expected Profit for Each Subject

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Figure 2 illustrates the projected profit for each subject, which spans from 1 to 25. The graph illustrates variations in anticipated profit, ranging from around 50 to 77 units. This fluctuation reflects variations in profitability among the participants.

The results also indicate a general tendency for subjects to stop between the 5th and 9th offers, with some outliers exhibiting later stopping points. The optimal stopping points reflect the subjects' attempts to balance the potential gains from waiting for better offers against the cumulative punishment incurred by delaying decisions.

The analysis of the study demonstrates significant variation in both expected profits and stopping points among the subjects, highlighting individual differences in risk preferences and decision-making strategies. The variability in these outcomes suggests that both the offer values and cumulative penalties played crucial roles in shaping the participants' decisions.

DISCUSSION

Influence of Cumulative Punishment

Cumulative punishment for decisions beyond the 5th stage had a significant impact on subjects' stopping behavior. As the penalty increased with each additional stage, many subjects adopted a more conservative strategy, stopping earlier in the sequence. This is evident in the data, where most subjects stopped around the 7th stage, and only a few extended their decisions beyond the 12th stage. The penalties acted as a deterrent to prolonged decision-making, mimicking real-life scenarios where delaying financial decisions incurs additional costs (e.g., transaction fees or opportunity costs). This aligns with the theoretical findings of Liu and Wei,[8] who showed that higher costs associated with delayed decisions lead to earlier stopping points.

Expected Profits and Risk Preferences

The expected profits for each subject varied significantly, ranging from 50,18 to 76,92. The subjects' stopping points and their sensitivity to the cumulative penalties largely explain this range. Subjects who stopped earlier tended to secure higher expected profits, as they avoided the increased penalties associated with later stages. Conversely, those who delayed decisions and incurred higher penalties experienced reduced profits. This behavior reflects individual differences in risk aversion, with more risk-tolerant individuals being willing to continue observing offers at the expense of lower profits.

Moreover, subjects who stopped earlier, often in the 6th or 7th stages, benefited from mid-to-high offer values (e.g., 78 or 87) without experiencing significant penalties. These participants were able to maximize their expected profits by balancing the opportunity to accept higher offers while mitigating the risk of cumulative punishment. On the other hand, more risk-tolerant subjects who waited for later offers saw lower profits, in part due to the accumulating penalties.

The distribution of probabilities for each offer did not appear to significantly influence the subjects' decisions, suggesting that risk preferences and penalties played a more prominent role. This is consistent with findings by Holt and Laury (2002), where individual variation in risk tolerance explained differences in economic decision-making.

Variability in Stopping Behavior

The observed stopping points show a high degree of variability among subjects, ranging from as early as the 1st stage to as late as the 19th stage. This pattern suggests that while most subjects exhibited risk-averse behavior, a few were willing to continue observing offers despite the penalties. For instance, the subjects who stopped in later stages often accepted lower offer values, expecting that subsequent offers would yield better opportunities. This overconfidence or tolerance for risk, noted by Barber and Odean,[17] ultimately led to lower profits for these individuals.

Implications for Financial Decision-Making

The study's findings have significant implications for financial decision-making. Financial advisors and investors can use these insights to better understand how risk aversion and cumulative costs influence investment strategies. By recognizing the tendency of risk-averse individuals to stop early and the impact of cumulative punishment, advisors can tailor their recommendations to align with clients' risk preferences and optimize their investment outcomes.

The results also suggest that investors should be mindful of the costs associated with prolonged decision-making and consider adopting strategies that mitigate these costs. Diversification, as suggested by Markowitz,[18] and appropriate timing of market entry and exit, as highlighted by Giannetti and Laeven,[19] are critical strategies for managing risk and optimizing financial returns.

Relevance to Sequential Decision-Making and Financial Markets

The study contributes to the broader literature on sequential decision-making and financial markets by demonstrating how risk aversion and cumulative costs shape stopping behavior. In dynamic financial markets, where investors must continuously evaluate new information and adjust their strategies, understanding these factors is crucial for making informed decisions.

The neural basis of risk aversion, as explored by Kuhnen and Knutson,[20] also provides a deeper understanding of the mechanisms underlying these behaviors. Recognizing that risk processing involves specific brain regions can inform the development of interventions and tools to help individuals make better financial decisions.

The study provides valuable insights into the role of risk aversion and cumulative punishment in sequential decision-making. By examining optimal stopping rules and expected profits, the research highlights the diverse risk preferences among individuals and the significant impact of cumulative costs on financial behavior. These findings contribute to the broader literature on financial decision-making and offer practical implications for investors and financial advisors.

CONCLUSION

This study has examined the optimal stopping rule and expected profits using a modified version of the Brickman principle, which incorporates cumulative punishment and probability factors. Our findings reveal significant variability in decision-making behavior among subjects, reflecting differences in risk tolerance and decision strategies. The results underscore the efficacy of the modified model in providing a more nuanced and accurate analysis of decision-making processes compared to traditional models.The analysis indicates that individuals with lower optimal stopping points tend to achieve higher expected profits, suggesting a more risk-averse approach. These individuals prefer to secure a guaranteed return earlier rather than risk waiting for potentially higher, but uncertain, offers. Conversely, those with higher stopping points often experience lower expected profits, indicating a higher tolerance for risk in the hope of achieving greater rewards. This dichotomy highlights the diversity in risk preferences and the importance of tailoring decision-making models to account for individual differences.

The incorporation of cumulative punishment and probability factors into the Brickman principle offers several advantages. It provides a more realistic framework for understanding decision-making under uncertainty by penalizing later decisions and adjusting for the likelihood of various offers. This approach captures both the immediate and long-term consequences of stopping decisions, offering a more comprehensive understanding of the decision-making process. Traditional models that overlook these factors may oversimplify the complexities involved, leading to less accurate predictions and suboptimal recommendations. The modified Brickman principle has significant practical implications for fields such as economics, psychology, and finance. In economics, it can enhance models of consumer behavior, investment decisions, and market dynamics by accounting for risk preferences and time-based penalties. In psychology, it offers a valuable tool for studying decision-making processes and developing interventions to improve decision quality. In finance, it can aid in the design of investment strategies and risk management practices by providing a better understanding of how individuals make decisions under uncertainty.

REFERENCES

1. Chow YS, Robbins H, Siegmund D. Great expectations: The theory of optimal stopping. Boston: Houghton Mifflin; 1971.

2. Kahneman D. Prospect theory: An analysis of decisions under risk. Econometrica. 1979;47:278.

3. Shefrin H, Statman M. The disposition to sell winners too early and ride losers too long: Theory and evidence. J Finance. 1985;40(3):777-90.

4. Brickman P. Optional stopping on ascending and descending series. Organ Behav Hum Perform. 1972;7(1):53-62.

5. Bertsekas DP. Dynamic programming and optimal control. 3rd ed. Belmont: Athena Scientific; 2005.

6. Pratt JW. Risk aversion in the small and in the large. Econometrica. 1964;32(1/2):122-36.

7. Arrow KJ. Essays in the theory of risk-bearing. Chicago: Markham Publishing Company; 1971.

8. Liu Y, Wei X. Incorporating risk preferences into optimal stopping rules. J Econ Dyn Control. 2013;37(5):985-97.

9. Odean T. Are investors reluctant to realize their losses? J Finance. 1998;53(5):1775-98.

10. Barberis N, Thaler R. A survey of behavioral finance. In: Constantinides GM, Harris M, Stulz RM, editors. Handbook of the economics of finance. Amsterdam: Elsevier; 2003. p. 1053-128.

11. Holt CA, Laury SK. Risk aversion and incentive effects. Am Econ Rev. 2002;92(5):1644-55.

12. Doherty NA. Integrated risk management: Techniques and strategies for managing corporate risk. New York: McGraw Hill Professional; 2000.

13. Caliendo M, Fossen FM, Kritikos AS. Risk attitudes of nascent entrepreneurs: New evidence from an experimentally validated survey. Small Bus Econ. 2009;32(2):153-67.

14. Byrnes JP, Miller DC, Schafer WD. Gender differences in risk taking: A meta-analysis. Psychol Bull. 1999;125(3):367-83.

15. Croson R, Gneezy U. Gender differences in preferences. J Econ Lit. 2009;47(2):448-74.

16. Hsee CK, Weber EU. Cross-national differences in risk preference and lay predictions. J Behav Decis Mak. 1999;12(2):165-79.

17. Barber BM, Odean T. Boys will be boys: Gender, overconfidence, and common stock investment. Q J Econ. 2001;116(1):261-92.

18. Markowitz H. Portfolio selection. J Finance. 1952;7(1):77-91.

19. Giannetti M, Laeven L. Flight home, flight abroad, and international credit cycles. Am Econ Rev. 2012;102(3):219-24.

20. Kuhnen CM, Knutson B. The neural basis of financial risk-taking. Neuron. 2005;47(5):763-70.

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